1. Validity. Make up an argument with the described premises and conclusion, or say why such an argument is impossible. (10 pts each)

(a) Valid, with one false premise, one true premise, and a false conclusion.

One example: Austin is the capital of Texas. Texas is the largest state. Therefore Austin is the capital of the largest state.

(b) Invalid, with two true premises, and a true conclusion.

One example: Grass is green. Snow is white. Therefore, I am a professor.

Another example: The Statue of Liberty is taller than my house. My house is taller than the Empire State Building. Therefore the Statue of Liberty is taller than the Empire State Building.

(c) Valid, with one true premise and a false conclusion.

This is impossible, because it is impossible for a valid argument to have true premises and a false conclusion.

2. Translations (10 pts each)

(a) Translate the following sentences from English into the formal language of Tarski’s World.

i. Either a or b is a medium tetrahedron.

\[ ((\text{Medium(a)} \land \text{Tet}(a)) \lor (\text{Medium(b)} \land \text{Tet}(b))) \]

ii. c and a are not both medium.

\[ \sim (\text{Medium(c)} \land \text{Medium(a)}) \]

iii. a is either a cube or a tetrahedron, and it is in the same row as d.

\[ (\text{Cube(a)} \lor \text{Tet}(a)) \land \text{SameRow(a,d)} \]
(b) Give ordinary English translations of the following sentences in the formal language of Tarski’s World.

i. \((\text{Large}(c) \land \text{Cube}(c)) \land (\text{Large}(a) \land \text{Tet}(a))\)

c is a large cube, and a is a large tet.

ii. \(\neg(\text{Cube}(a) \lor \text{Tet}(a))\)

a is neither a cube nor a tet.

iii. \(\text{LeftOf}(a,b) \land \neg \text{BackOf}(a,c)\)

a is to the left of b, but it is not behind c.

3. Say whether each of the following arguments is valid. If it is valid, show this using a truth table. If it is invalid, show this by describing a situation that would be a counterexample. (20 pts each)

(a) Cube(a). Large(a) \lor Small(a). Therefore, \(((\text{Large}(a) \land \text{Cube}(a)) \lor (\text{Small}(a) \land \text{Cube}(a)))\).

(b) Cube(a) \lor Cube(b). \neg (\text{Cube}(a) \land \text{Small}(a)). Therefore, \text{Small}(a) \lor \text{Cube}(b).

Consider a possibility where a is a large cube, but b is a tetrahedron.
The first premise would be true, because a is a cube.
The second premise would be true, because a is not a small cube.
The conclusion would be false, because a is not small and b is not a cube.

Since this is a possibility where the premises are true and the conclusion is false, the argument is not valid.

4. Consider the following table. What is the probability of A \lor C? What is the probability of A \lor C given \neg B? Is the argument, “\neg B, therefore A \lor C” a good one? How good? (20 pts)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>.2</td>
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<td>T</td>
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</tbody>
</table>

AvC is true in all rows but the sixth and eighth, so its probability is .2 + .1 + .2 + .1 = .6

\neg B is true in rows 3, 4, 7, 8, so it has probability .5

Out of that .5, AvC has .1 + .2 + .1 = .4.

Thus, the probability of AvC given \neg B is .4 / .5 = .8

Thus, the premise does not change the probability of the conclusion of this argument, so it provides no support at all for the conclusion, and the argument is not good.